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Effect of the Presence of a Heated Circular Obstacle on Natural Convection inside a Square Enclosure

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ABSTRACT

Finite element method is used to solve the two dimensional governing mass, momentum and energy equations for steady state, natural convection problem inside a square enclosure. The enclosure consists of adiabatic top and bottom walls, cool left and right walls and a uniformly heated circular solid body located somewhere inside the enclosure. The aim of this study is to describe the effect of the presence of circular heated obstacle on natural convection. The investigations are conducted for different values of Rayleigh number (Ra), obstacle diameter (D) and location of obstacle Centre (Cx, Cy) inside the enclosure. Various results such as streamlines, isotherms, heat transfer rate in terms of the average Nusselt number and average fluid temperature inside the enclosure are presented for different parameters. The results indicate that the average Nusselt number at the heated surface and average temperature of the fluid inside the enclosure are strongly dependent on the configuration of the system under different geometrical and physical conditions.

Keywords: Natural convection, finite element method, square enclosure, circular obstacle

Nomenclature

c_p	Specific heat at constant pressure	x, y	Cartesian coordinates
g	gravitational acceleration $[ms^{-2}]$	X, Y	dimensionless Cartesian coordinates
h	Convective heat transfer coefficient $[Wm^{-2}K^{-1}]$	(C_x, C_y)	circular obstacle centre
k	Thermal conductivity of fluid $[Wm^{-1}K^{-1}]$	Nu	average Nusselt number
T	Dimensional temperature $[K]$	P	dimensionless pressure
ΔT	Dimensional temperature difference $[K]$	Pr	Prandtl number
u, v	Dimensional velocity components $[ms^{-1}]$	Ra	Rayleigh number
U, V	Dimensionless velocity components	D	Diameter of circular obstacle $[m]$
p	Dimensional pressure $[Nm^{-2}]$		

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INTRODUCTION

Natural convection is an essential mechanism of heat transfer in modern technology due to its wide applications. Various numerical and experimental investigations are done without and with obstacle in the various shaped enclosure because these geometries have practical applications in engineering and industrial fields. Some mentionable fields are cooling of electronic devices, air conditioning, thermal design of building, design of solar collectors, chemical processing equipment, drying technologies etc. Literature review shows that various investigations are done on the mechanism of natural convection in a square enclosure containing various fluids with different geometrical parameters and boundary conditions. Parvin and Nasrin analyzed the flow and heat transfer characteristics for MHD free convection in an enclosure with a heated obstacle. They showed that the free convection parameter Ra and the diameter of circular body have significant effect on the flow and temperature fields. Chowdhury et al. investigated the natural convection in porous triangular enclosure with a circular obstacle in presence of heat generation. They showed that the fluid flow and temperature field strongly depend on the presence of the circular body and the average Nusselt number significantly worse with increasing both heat generation and size of the obstacle. House et al. investigated the effect of a centered, square heat conducting body on natural convection in a vertical enclosure. They showed that heat transfer across the cavity enhanced or reduced by a body with a thermal conductivity ratio less or greater than unity. Kandaswamy et al. investigated natural convection in a square cavity in presence of heated plate. They showed that for the increasing value of Gr heat transfer rate increases in both vertical and horizontal situation. As the aspect ratio of heated thin plate increases the heat transfer also increases. Mousa investigated the modeling of laminar buoyancy convection in a square cavity containing an obstacle. They found that in case of low Rayleigh numbers (102 – 104), the rate of heat transfer decreases when the aspect ratio of the adiabatic square obstacle increases. In case of relatively high Rayleigh numbers (105 –106), the maximum heat transfer rate increases when the aspect ratio of the adiabatic square obstacle increases. Rahman et al. investigated the effect of presence of a heat conducting horizontal square block on mixed convection inside a vented square cavity by using finite element method. They found that the block size and location have significant effect on both the flow and thermal fields but the solid-fluid thermal conductivity ratio has insignificant effect on the flow field. Roslan et al. investigated the natural convection in an enclosure containing a sinusoidal heated cylindrical source. They showed that heat transfer rate tends to increase by oscillating the source temperature signal. The maximum heat transfer augmentation is obtained at about frequency 25π to 30π for high heating amplitude and a moderate source radius. Rahman et al. studied numerically the mixed convection in a square cavity with a heat conducting square cylinder at different locations. They showed that the flow field and temperature distributions inside the cavity are strongly dependent on the Richardson numbers and the position of the inner cylinder. Saleh et al. investigated the natural convection heat transfer in a nano fluid-filled trapezoidal enclosure. They found that the structure of fluid flow within the enclosure depends upon Grashof number, inclination angle of slopping wall and nanoparticles concentration and type. Uddin et al. investigated the natural convection flows in a trapezoidal enclosure with isoflux heating from below. They showed that the average Nusselt number increases with the increase of Rayleigh number and the effect of the sidewall inclination angle on heat transfer is significantly reduced at higher Rayleigh number.

Physical model

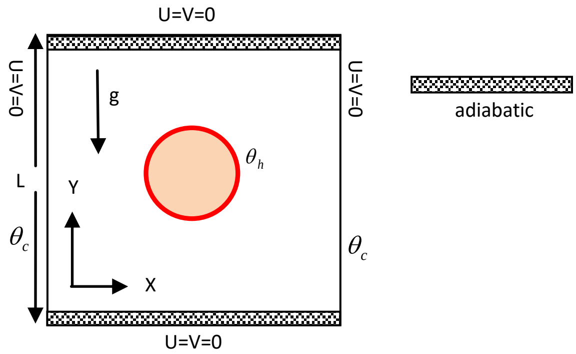

Fig 1. Schematic view of the enclosure considered in present study

Mathematical formulation

In the present problem, it is assumed that the flow is steady, two-dimensional, laminar, and incompressible and there is no viscous dissipation. The gravity force acts in the vertically downward direction, fluid properties are constant and fluid density variations are neglected except in the buoyancy term (Boussinesq approximation) and radiation effect is neglected. Under the usual Boussinesq approximation, the governing equations for the present problem can be described in dimensionless form by the following equations.

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ra\Pr\theta$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right)$$

The dimensionless variables are defined as:

$$X=\frac{x}{L},\qquad Y=\frac{y}{L},\qquad U=\frac{u}{U_0},\qquad V=\frac{v}{U_0},\quad P=\frac{p}{\rho {U_0}^2},\qquad \theta=\frac{T-T_c}{T_h-T_c}$$

The governing parameters in the preceding equations are the Rayleigh number $Ra=\frac{g\beta(T_h-T_c)L^3}{\upsilon\alpha}$ and Prandtl number $\Pr=\frac{\upsilon}{\alpha}$

The associated dimensionless boundary conditions are as follows:

$P = 0$ at all boundaries

$$U(X,0) = U(X,1) = U(0,Y) = U(1,Y) = 0$$

$$V(X,0) = V(X,1) = V(0,Y) = V(1,Y) = 0$$

$$\theta(0,Y) = \theta(1,Y) = 0$$

$$\frac{\partial \theta(X,0)}{\partial Y} = \frac{\partial \theta(X,1)}{\partial Y} = 0$$

At the circular body surface $U(X,Y) = V(X,Y) = 0, \quad \theta(X,Y) = 1$

The average Nusselt number at the heated wall of the circular body surface is defined as $Nu = -\frac{1}{L_s}\int_0^{L_s/L} \frac{\partial \theta}{\partial n} dS,$ where $\frac{\partial \theta}{\partial n} = \sqrt{\left(\frac{\partial \theta}{\partial X}\right)^2 + \left(\frac{\partial \theta}{\partial Y}\right)^2}$ and S is the non-dimensional coordinate along the circular surface. The bulk average temperature in the enclosure is defined as $\theta_{av} = \int \frac{\theta}{S} dS$

NUMERICAL TECHNIQUE

The numerical procedure used in this work is based on the Galerkin weighted residual method of finite element formulation. The application of this technique is well described by Taylor and Hood and Dechaumphai. In this method, the solution domain is discretized into finite element meshes, which are composed of non-uniform triangular elements. Then the nonlinear governing partial differential equations (i.e. mass, momentum and energy equations) are transferred into a system of integral equations by applying Galerkin Residual method. The integration involved in each term of these equations is performed by using Gauss's quadrature method. The nonlinear algebraic equations so obtained are modified by imposition of boundary conditions. These modified nonlinear are transferred into linear algebraic equations by Newton-Raphson iteration. Finally, these linear equations are solved by using Triangular Factorization method.

Numerical validation:

To validate the present numerical code, the results for natural convection flow around a heated circular obstacle placed somewhere in the enclosure have been compared with those obtained by Parvin and Nasrin. The comparison of the results obtained from present code with those of Parvin and Nasrin is demonstrated for two different Rayleigh numbers $Ra = 10^3$ and $Ra = 10^5$ at *Ha=50, D=0.25, Pr=0.7* in fig.2. As seen from these figures the obtained results show very good agreement.

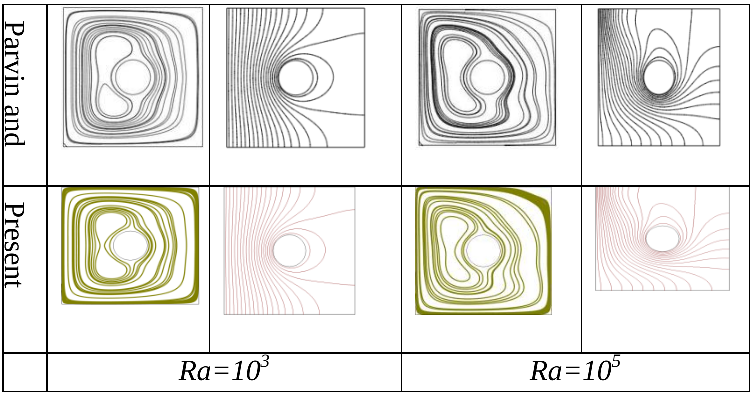

Fig 2. Comparison of streamlines and isotherms between present model (lower) and the model of Parvin and Nasrin (upper) at $Ha=50$, $D=0.25$, $Pr=0.7$

RESULTS AND DISCUSSION

A numerical study has been performed through finite element method to analyze the laminar natural convection heat transfer and fluid flow in a square cavity containing a horizontal circular obstacle. Effect of the dimensionless parameters such as Rayleigh number *(Ra)*, dimensionless obstacle diameter *(D)*, and the centre position of heated obstacle (C_x, C_y) on heat transfer and fluid flow have been analyzed. The result is presented in two sections. The first section has focused on flow and temperature fields, which contains the streamlines and isotherms for different cases. Heat transfer including average Nusselt number at heated surface and average fluid temperature in the enclosure has been discussed in the following section. The ranges of *Ra* and *D* in this investigation vary from $Ra=10^3$ to 10^7 and $D=0.10$ to 0.60. Centre position (C_x, C_y) of heated obstacle are considered at *(0.25, 0.25)*, *(0.25, 0.75)*, *(0.75, 0.25)* and *(0.75, 0.75)* while the Prandtl number *Pr* is kept fixed at *0.71*.
The influence of Rayleigh number *Ra* (from $Ra=10^3$ to 10^7) and obstacle diameter *D* (from 0.10 to 0.60) on streamlines for the present configuration at $Pr=0.71$ and centre position (C_x, C_y) *at (0.5, 0.5)* has been demonstrated in Fig.3. Fluid flow has been affected by the buoyancy force created by the temperature difference of circular body and vertical side walls. For $Ra=10^3$ at $D=0.10$ it has created two separate vortices near two cold vertical walls. As *D* increases to 0.30, the vortices become larger creating two more inner vortices. Further increase of *D* enlarges the size of inner and outer vortices occupying the whole enclosure. For $Ra=10^5$, the streamlines behave almost the same as that of $Ra=10^3$. For $Ra=10^7$, vortices accumulate in the vicinity of upper wall. With the increasing value of *D* the vortices gradually occupy the whole enclosure creating more inner vortices. The influence of Rayleigh number *Ra* (from $Ra=10^3$ to 10^7) and obstacle diameter *D* on temperature field for the present configuration at $Pr=0.71$ and centre position (C_x, C_y) *at (0.50, 0.50)* has been demonstrated in Fig.4. The high temperature region exists in the two vertical sides of the circular obstacle and the isothermal lines are linear and parallel to the vertical walls for $Ra=10^3$.

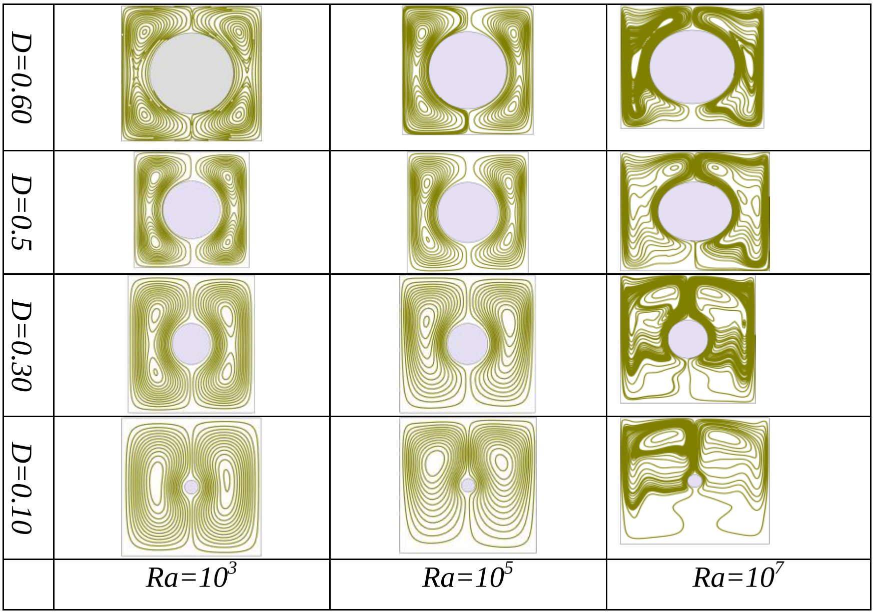

Fig 3. Streamlines for different values of *Ra* and *D* with *Pr=0.71* and centre at *(0.50, 0.50)*

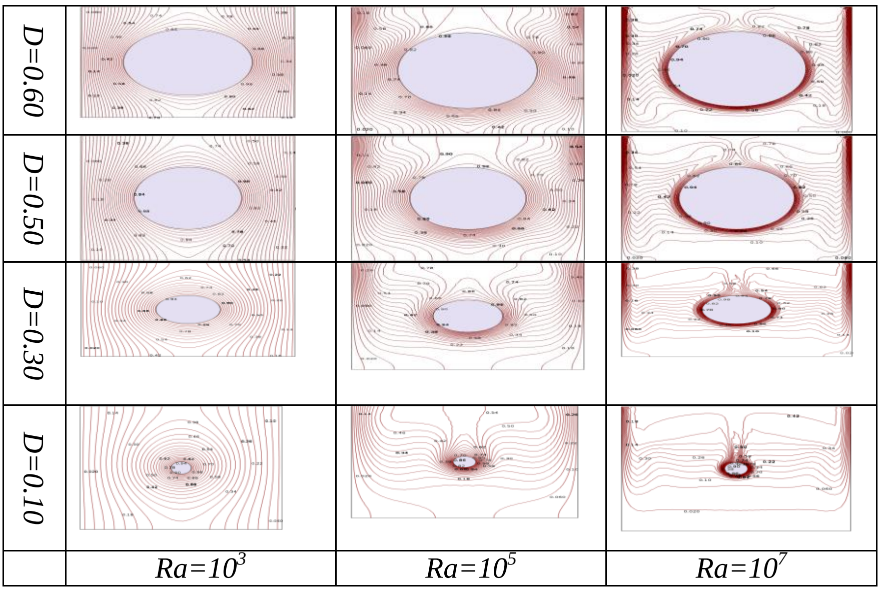

Fig 4. Isotherms for different values of *Ra* and *D* with *Pr=0.71*, centre at *(0.5, 0.5)*

With the increasing value of *Ra* the isothermal lines become more curved from both the sides. Further increase of *Ra* creates a plume in the upper side of obstacle and isothermal lines become almost parallel to horizontal walls changing their direction. Besides this, the isothermal lines are concentrated near the heated obstacle with the increasing value of *D*. The effect of obstacle location on streamlines for the present configuration at $Pr=0.71$ and $D=0.30$ has been demonstrated in Fig.5. When the center of inner obstacle is situated at *(0.25, 0.25)* as shown in bottom row of Fig.5, for $Ra=10^3$ a small vortex in the left top side and a large vortex in the right side are created. As *Ra* increases to 10^5 the small vortex becomes larger. At $Ra=10^7$ the concentration of both vortices increases and occupies the whole enclosure. At the same time the small vortex becomes a tri-cellular vortex.

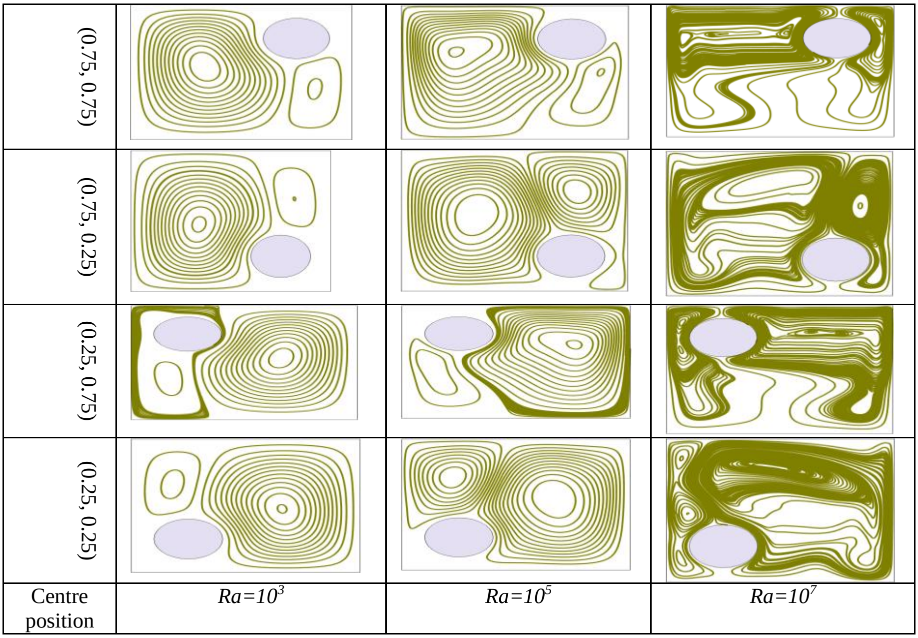

Fig 5. Streamlines for different values of *Ra* and different position of obstacle centre with $Pr=0.71$ *and* $D=0.30$

The behavior of streamlines for other locations of center is almost the same. The mentionable change is for all the cases vortices are created in the opposite side of circular obstacle i.e. in the larger unoccupied space of the enclosure. The effect of obstacle location on isotherms for the present configuration at $Pr=0.71$ and $D=0.30$ has been demonstrated in Fig.6. When the center of inner obstacle is situated at *(0.25, 0.25)* as shown in bottom row of Fig.6. For $Ra=10^3$ and various locations of obstacle centre, the isotherms nearer to obstacle are found to concentrate the heated obstacle and the isotherms which are far from the obstacle are almost parallel to the vertical walls. As *Ra* increases to 10^5, the concentration of isothermal lines in the vicinity of heated obstacle increases. At the same time the bending of isothermal lines increases and created a plume shape in the upper side of the obstacle. At $Ra=10^7$ the bending of the isothermal lines and concentration near the heated obstacle both are increased and they form an unstable shape.

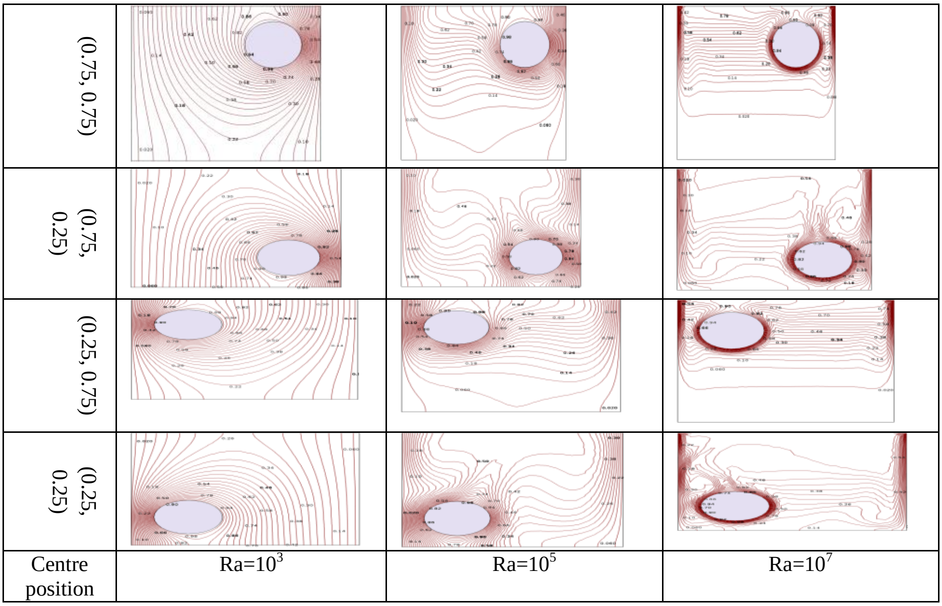

Fig 6. Isotherms for different values of *Ra* and different position of obstacle center with *Pr=0.71 and D=0.30*

In order to evaluate how the obstacle size affects the heat transfer along the heated surface, the average Nusselt number is plotted as a function of Rayleigh number as shown in Fig. 7(a) for four different obstacle sizes (*D=0.10*, *0.30*, *0.50* and *0.60*) while *Pr=0.71* and obstacle center at (*0.5, 0.5*).

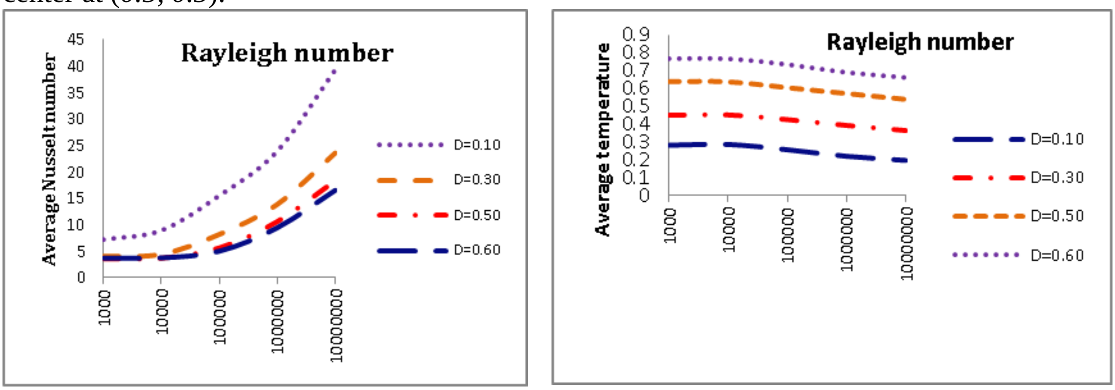

Fig 7. (a) average Nusselt number (b) average temperature with *Pr=0.71* and centre at (*0.5, 0.5*)

It is observed that Nu decreases with the increasing value of diameter and increases with the increasing of *Ra*. Also Fig. 7(b) depicts the average temperature of fluid as a function of Rayleigh number *Ra* while *Pr=0.71* and obstacle center at (*0.5, 0.5*). It is observed that fluid temperature increases with the increasing value of diameter but it decreases slightly with the increasing value of *Ra*.

CONCLUSION

A numerical investigation is made for steady-state, incompressible, laminar natural convection flow in a square enclosure containing a heated circular obstacle. Results are obtained for wide ranges of parameters Rayleigh number *(Ra),* dimensionless obstacle diameter *(D)* and the location of circular obstacle (C_x, C_y). The major findings have been summarized as follows:

- The free convection parameter *Ra* affects strongly the streamline distribution in the enclosure. As a result buoyancy force induced circulation cell increases with the increasing value of *Ra*.
- The diameter of the circular obstacle has a significant effect on the flow and temperature fields. The recirculation cell in the streamlines increases and the isothermal lines near the heated surface become denser with the increasing value of *D.*
- Locations of the circular obstacle do not affect the flow and thermal fields significantly.
- The average Nusselt number *Nu* at the circular body surface is enhanced for larger values of *Ra* and lower values of *D.*

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