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Shear Rates Calculation on Natural Convection of Non-Newtonian Fluid over a Horizontal Circular Cylinder with Uniform Surface Heat Fluxes

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ABSTRACT
Natural convection of a laminar two-dimensional boundary-layer flow of non-Newtonian fluids over a horizontal circular cylinder with the heating condition of uniform surface heat flux has been studied using a modified power-law viscosity model. In this model, there are no unrealistic limits of zero or infinite viscosity; consequently, no irremovable singularities are introduced into boundary-layer formulations for such fluids. Therefore, the boundary-layer equations can be solved numerically by using marching order implicit finite difference method with double sweep technique. Numerical results are mostly presented for the case of shear-thinning as well as shear thickening fluids in terms of the shear rates. Fluid velocity and temperature distributions, shear stresses and rate of heat transfer in terms of the local skin-friction and local Nusselt number respectively are also presented.

1. INTRODUCTION

Natural convection laminar flow of non-Newtonian power-law fluids from a horizontal circular cylinder with uniform heat flux presents an important role in numerous engineering applications those are related with pseudo-plastic fluids. The pseudo-plastic fluid is characterized by a constant viscosity at very low shear rates, a viscosity which decreases with shear rate at intermediate shear rates and an apparently constant viscosity at very high shear rate. The interest in skin-friction and heat transfer problems involving power-law non-Newtonian fluids has grown in the past half-century. An excellent research on non-Newtonian fluids was given by Boger [1]. Acrivos [2] was the first to consider boundary-layer flows for such non-Newtonian fluids. Since then, a large number of papers have been published, due to their wide relevance in pseudo-plastic fluids like chemicals, foods, polymers, molten plastics and petroleum production and various natural phenomena.

An entire assessment of these literatures was impractical; however, selected papers are listed here to provide starting points for a broader literature search [3-15]. In the boundary-layer study, they used the traditional power-law viscosity correlation that viscosity becomes infinite for small shear rates or vanishes for the limits of large shear rates, which are giving the unrealistic physical results. Because an infinite viscosity corresponds to solids and no frictionless fluid has ever been found, a partial set of measured viscosity shear relations is not sufficient for a boundary-layer study.

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In recent times proposed modified power-law correlation is sketched for various values of power index *n* in Fig. 2 and this model is formulated based on the available experimental data for the non-Newtonian fluids (see Boger [1]). It is clear that the new correlation does not contain the physically unrealistic limits of zero and infinite viscosity displayed by traditional power-law correlations [2]. The modified power-law, in fact, fits measured viscosity data well. The constants in the proposed model can be fixed with available measurements and are described in detail in Yao and Molla [16]. The boundary-layer formulation on a flat plate is described and numerically solved for non-Newtonian fluid in Yao and Molla [16, 17] and the associated heat transfer for two different heating conditions is reported in Molla and Yao [18, 19] for shear-thinning fluid. The boundary-layer formulation along an isothermal horizontal circular cylinder is also described and numerically solved for non-Newtonian fluid in Bhowmick and Molla [20] for the case of shear-thinning as well as shear thickening fluids. In this investigation, the behavior of both shear-thinning and shear-thickening fluids on the natural convection laminar flow with uniform heat flux along a horizontal circular cylinder are studied by choosing the power-law index as n (= 0.6, 0.8, 1.0, 1.2, 1.4) to fully demonstrate the performance of various non-Newtonian fluids.

It is precious to message that the soundness of the laminar boundary-layer theory has been well established for nearly a century. Power-law correlations have also been used for almost half a century. It is well known that they can correlate a major part of the available data. The recently proposed modified power-law simply modifies the power-law to fit available data better at its two ends, because a power-law model is an undeviating model that is used to fit experimental data.

2. FORMULATION OF THE PROBLEM

A two-dimensional steady laminar natural convection boundary-layer of a non-Newtonian fluid over a horizontal circular cylinder of radius 'a' with uniform surface heat flux and a distributed heat source of the form $g\beta(T - T_\infty)$ has been considered. The viscosity depends on shear rate and is correlated by a modified power-law. We consider shear-thinning and shear-thickening situations of non-Newtonian fluids. It is assumed that a surface heat flux q_w is applied to the cylinder; T_∞ is the ambient temperature of the fluid and T is the temperature of the fluid. The configuration considered is as shown in Fig. 1.

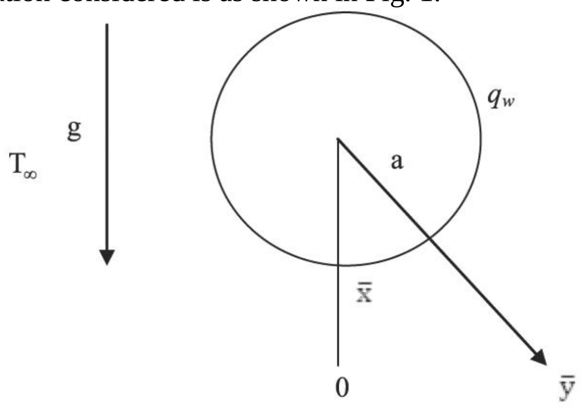

Fig. 1 The flow model and coordinate system.

Under the above assumptions, the boundary-layer equations governing the flow and heat transfer are

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{1}$$

$$\rho\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = \frac{\partial}{\partial \bar{y}}\left(\mu \frac{\partial \bar{u}}{\partial \bar{y}}\right) + \rho g \beta (T - T_\infty)\sin\left(\frac{\bar{x}}{a}\right) \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial \bar{y}^2} \tag{3}$$

Where $\bar{u}$ and $\bar{v}$ are velocity components along the $\bar{x}$ and $\bar{y}$ axes, ρ is the fluid density, μ is the dynamic viscosity of the fluid in the boundary-layer region, g is the acceleration due to gravity, β is the coefficient of thermal expansion, k is the thermal conductivity and C_p is the specific heat at constant pressure. The kinematic viscosity $\nu = \mu/\rho$ is correlated by a modified power-law, which is

$$\nu = \frac{K}{\rho}\left|\frac{\partial \bar{u}}{\partial \bar{y}}\right|^{n-1} \text{ for } \bar{\gamma}_1 \le \left|\frac{\partial \bar{u}}{\partial \bar{y}}\right| \le \bar{\gamma}_2 \tag{4}$$

The constants $\bar{\gamma}_1$ and $\bar{\gamma}_2$ are threshold shear rates, which are given according to the model of Boger [1], K is the dimensional constant, for which dimension depends on the power-law index n. The values of these constants can be determined by matching with measurements. Outside of the preceding range, viscosity is assumed to be constant; its value can be fixed with data given in Fig. 2.

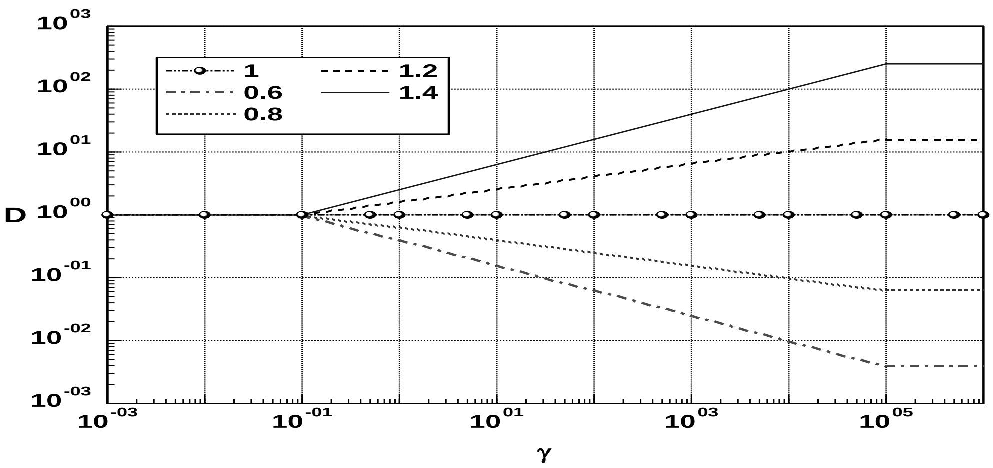

Fig. 2 Modified power-law correlation for the power-law index *n* (= 0.6, 0.8, 1.0, 1.2, 1.4) while $\gamma_1 = 0.1$ and $\gamma_2 = 10^5$.

The boundary conditions for the present problems are

$$\bar{u} = 0, \quad \bar{v} = 0, \quad -k\frac{\partial T}{\partial \bar{y}} = q_w \qquad at \quad \bar{y} = 0 \tag{5a}$$

$$\bar{u} \to 0, \qquad T \to T_\infty \qquad as \quad \bar{y} \to \infty \tag{5b}$$

We introduce non-dimensional dependent and independent variables according to,

$$x = \frac{\bar{x}}{a}, \quad y = \frac{\bar{y}}{a}Gr^{1/5}, \quad u = \bar{u}\frac{a}{\nu_1}Gr^{-2/5}, \quad v = \bar{v}\frac{a}{\nu_1}Gr^{-1/5}$$

$$\theta = \frac{T - T_\infty}{(q_w a / k)}, \quad Gr = \frac{g\beta q_w a^4}{k\nu_1^2}, \quad \nu_1 = \frac{\mu_1}{\rho}, \quad D = \frac{\nu}{\nu_1}, \quad \Pr = \frac{\nu_1 \rho C_p}{k} \tag{6}$$

Where, ν_1 is the reference viscosity at $\bar{\gamma}_1$, θ is the non-dimensional temperature of the fluid, *Gr* is the Grashof number and Pr is the Prandtl number. Using equation (6) in equations (1-4) we get the following non-dimensional equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{7}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(D\frac{\partial u}{\partial y}\right) + \theta \sin x \tag{8}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} \tag{9}$$

$$D = \frac{K}{\rho \nu_1}\left|\frac{\nu_1}{a^2}Gr^{3/5}\frac{\partial u}{\partial y}\right|^{n-1}$$

$$= \frac{K}{\rho \nu_1}\left(\frac{\nu_1}{a^2}\right)^{n-1}\left[\frac{g\beta q_w a^4}{k\nu_1^2}\right]^{-\frac{3(n-1)}{5}}\left|\frac{\partial u}{\partial y}\right|^{n-1} = C\left|\frac{\partial u}{\partial y}\right|^{n-1} \tag{10}$$

The length scale associated with the non-Newtonian power-law is

$$a = C^{\frac{5}{2(n-1)}} \left[\left(\frac{K}{\rho} \right)^{\frac{5}{2}} \frac{1}{\nu_1^{(n+4)/2}} \right]^{\frac{1}{(1-n)}} \left(\frac{k}{g\beta\, q_w} \right)^{\frac{3}{2}} \tag{11}$$

The corresponding boundary conditions are

$$u = 0, \quad v = 0, \quad \frac{\partial \theta}{\partial y} = -1 \qquad at \quad y = 0 \tag{12a}$$

$$u \to 0, \quad \theta \to 0 \quad as \quad y \to \infty \tag{12b}$$

Now we introduce the parabolic transformation:

$$X = x, \quad Y = y, \quad U = \frac{u}{x}, \quad V = v, \quad \theta = \theta \tag{13}$$

Substituting variable (13) into equations (7-10) leads to the following equations:

$$X \frac{\partial U}{\partial X} + U + \frac{\partial V}{\partial Y} = 0 \tag{14}$$

$$XU \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} + U^2 = D \frac{\partial^2 U}{\partial Y^2} + \frac{\partial U}{\partial Y} \frac{\partial D}{\partial Y} + \frac{\theta \sin X}{X} \tag{15}$$

$$XU \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial Y^2} \tag{16}$$

$$D = \begin{cases} 1, & |\gamma| \le \gamma_1 \\ \left| \dfrac{\gamma}{\gamma_1} \right|^{n-1}, & \gamma_1 \le |\gamma| \le \gamma_2 \\ \left| \dfrac{\gamma_2}{\gamma_1} \right|^{n-1}, & \gamma_2 \le |\gamma| \end{cases} \qquad \text{where, } \gamma = X \frac{\partial U}{\partial Y} \tag{17}$$

The correlation (17) is a modified power-law correlation first presented by Yao and Molla [16]. This correlation describes that if the shear rate $|\gamma|$ lies between the threshold shear rates γ_1 and γ_2, then the non-Newtonian viscosity, *D*, varies with the power-law of *γ*. On the other hand, if the shear rate $|\gamma|$ does not lie within this range, then the non-Newtonian viscosities are different constants, as shown in Fig. 2. This is a property of many measured viscosities.

Equation (14-16) can be solved by marching downstream with the leading edge condition satisfying the following differential equations, which are the limits of equations (14-16) as $X \to 0$.

$$U + \frac{\partial V}{\partial Y} = 0 \tag{18}$$

$$V\frac{\partial U}{\partial Y} + U^2 = D\frac{\partial^2 U}{\partial Y^2} + \frac{\partial U}{\partial Y}\frac{\partial D}{\partial Y} + \theta \tag{19}$$

$$V\frac{\partial \theta}{\partial Y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} \tag{20}$$

The corresponding boundary conditions are

$$U = 0, \quad V = 0, \quad \frac{\partial \theta}{\partial y} = -1 \qquad \text{at} \quad Y = 0 \tag{21a}$$

$$U \to 0, \quad \theta \to 0 \quad \text{as} \quad Y \to \infty \tag{21b}$$

Equations (14-16) and (18-20) are discretized by a central-difference scheme for the diffusion term and a backward-difference scheme for the convection terms. Finally, we get an implicit tri-diagonal algebraic system of equations, which can be solved by a double-sweep technique. The normal velocity is directly solved from the continuity equation. The computation is started at *X*=0 and marches to downstream to *X*=3.1416. After several test runs, converged results are obtained by using $\Delta X = 0.0025$ and $\Delta Y = 0.005$.

In practical applications, the physical quantities of principle interest are the local skin-friction coefficients C_f and the local Nusselt number *Nu*, which are

$$C_f \left[Gr/4X\right]^{1/5} = \left[D\frac{\partial U}{\partial Y}\right]_{Y=0} \tag{22}$$

$$Nu[Gr/4X]^{-1/5} = \frac{1}{\theta(X,0)} \tag{23}$$

3. RESULTS AND DISCUSSION

The numerical results are presented for the non-Newtonian power-law of shear-thinning fluids (n = 0.6 and 0.8) and the shear-thickening fluids (n = 1.2 and 1.4) as well as the Newtonian case (n = 1) while Prandtl number, Pr = 10 and 50. Based on the experimental data of Boger [1] the thresholds shears γ_1 and γ_2 have been chosen as 0.1 and 10^5, respectively. The obtained results include the shear rates, velocity and temperature distribution, and the wall shear stress in terms of the local skin-friction coefficient, $C_f(Gr/4X)^{1/5}$ and the rate of heat transfer as a form of the local Nusselt number, $Nu(Gr/4X)^{-1/5}$ for the wide range of the power-law index n (= 0.6, 0.8, 1.0, 1.2, 1.4). The singularity experienced at the leading edge for the traditional power-law correlation has been successfully removed by using the modified power-law correlation. Since the shear-stress at the leading edge is inversely proportional to $(Gr/4X)^{1/5}$ and so is infinite there, $D=(\gamma_2/\gamma_1)^{n-1}$ at the leading edge.

Figures 3(a-f) show the corresponding shear rates for Pr = 10 and 50, respectively. For the shear-thinning fluids (n = 0.6 and 0.8), the boundary-layer thickness decreases more at the down stream region than for the shear-thickening fluids (n = 1.2 and 1.4). The boundary-layer thickness for Pr = 50 is less than half of the boundary-layer for Pr = 10. All the figures for Pr = 10 are comparatively smooth at Y axis than for Pr = 50. At X = 3 for Pr = 50, both the fluids (shear-thinning and shear-thickening) are same after the leading edge to down stream regions, but for Pr = 10 at X = 3 the fluids are showing different up to down stream regions. Shear rates of the fluids are maximum at the middle (X = 2) of the cylinder. Again, at X = 1 the shear rates are higher than at X = 3. Shear-thinning fluids are larger than shear-thickening fluids. Again, the boundary-layer at the initial wall of the shear rates is thinner than the middle or last position of the cylinder.

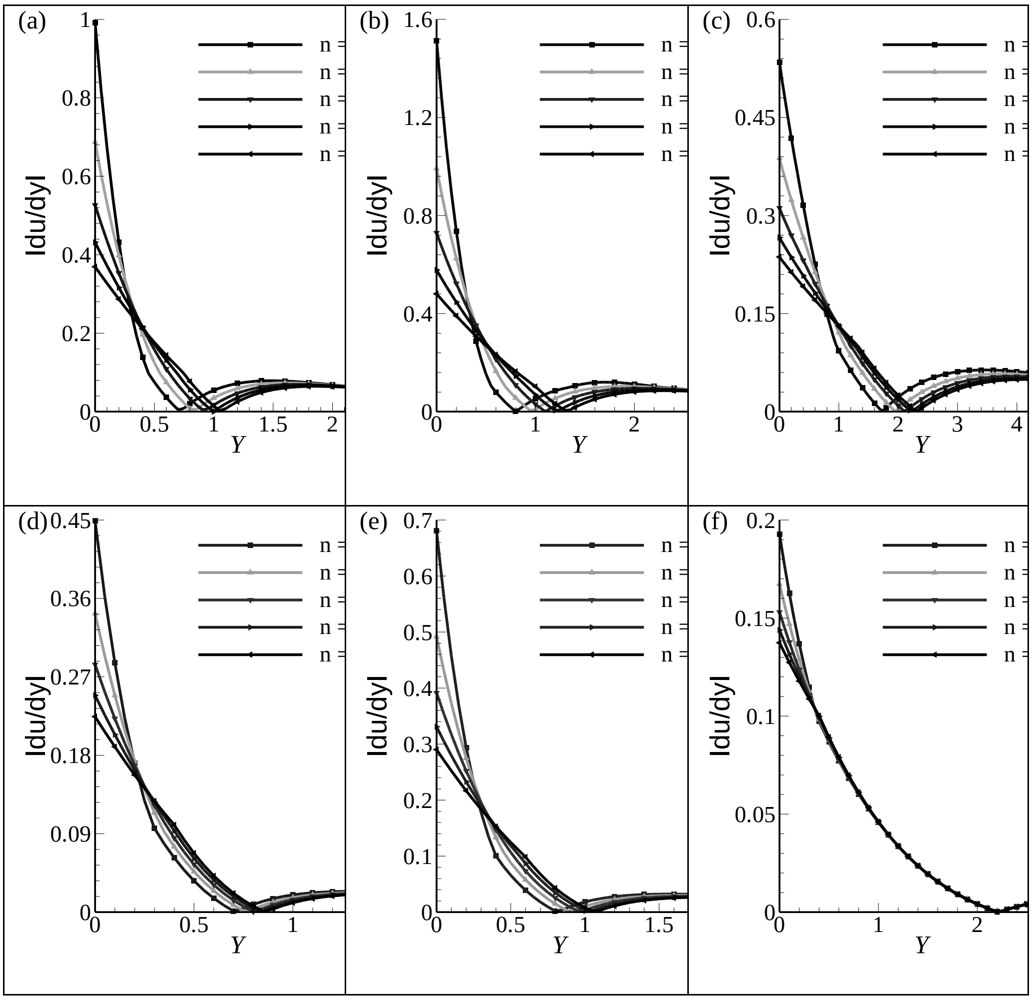

Fig. 3 Shear rates for different n at (a) X = 1, (b) X = 2, (c) X = 3 at Pr = 10 and (d) X = 1, (e) X = 2, (f) X = 3 at Pr = 50.

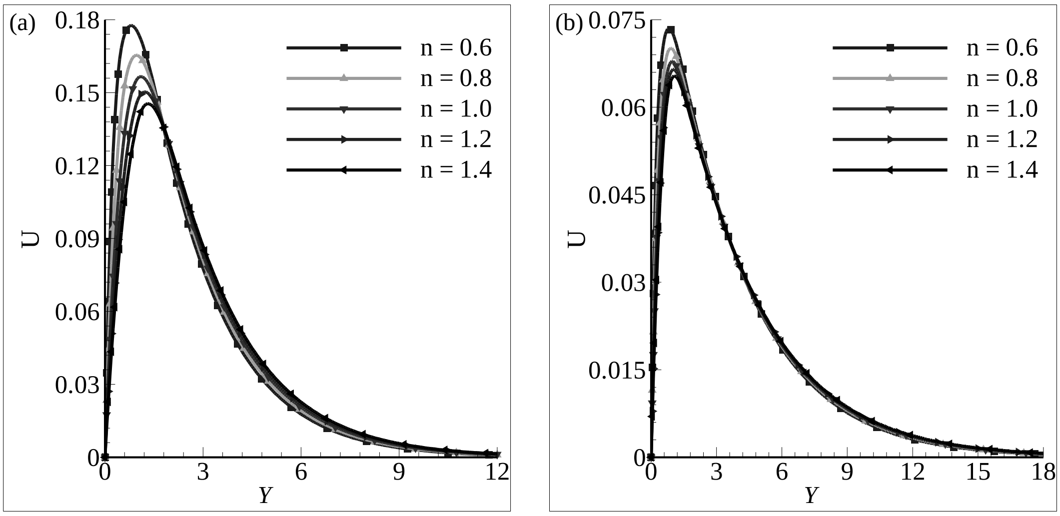

Fig. 4 Velocity distribution for different n at $X = 2$ for (a) Pr = 10 and (b) Pr = 50.

The velocity distribution as a function of Y at $X = 2$ for the different power-law indices (n = 0.6, 0.8, 1.0, 1.2, 1.4) are presented in Figs. 4(a) for Pr = 10 and 4(b) for Pr = 50, respectively. Fig. 4 shows that for shear-thinning fluids (n=0.6 and 0.8), the velocity increases due to the decrease of viscosities at the down stream region; consequently, the boundary–layer is thinned. On the other hand, for shear-thickening fluids (n=1.2 and 1.4), the velocity decreases slowly and the boundary-layer is thickened as the fluid becomes more viscous. We may conclude that for Pr = 50, the fluid velocity is smaller than that for Pr = 10 and the boundary-layer thickness is larger for Pr = 50 than that for Pr = 10.

The corresponding temperature distribution are plotted for Pr = 10 and 50 in Figs. 5(a) and 5(b), respectively. At the leading edge temperature of shear-thickening fluids is higher than shear-thinning fluids. For both of these Prandtl numbers, at the down stream region, in the case of shear-thinning fluids, the variation of temperature in the boundary-layer is smaller than that of the shear-thickening non-Newtonian fluids. As expected, the thermal boundary-layer is thinner for larger Prandtl numbers.

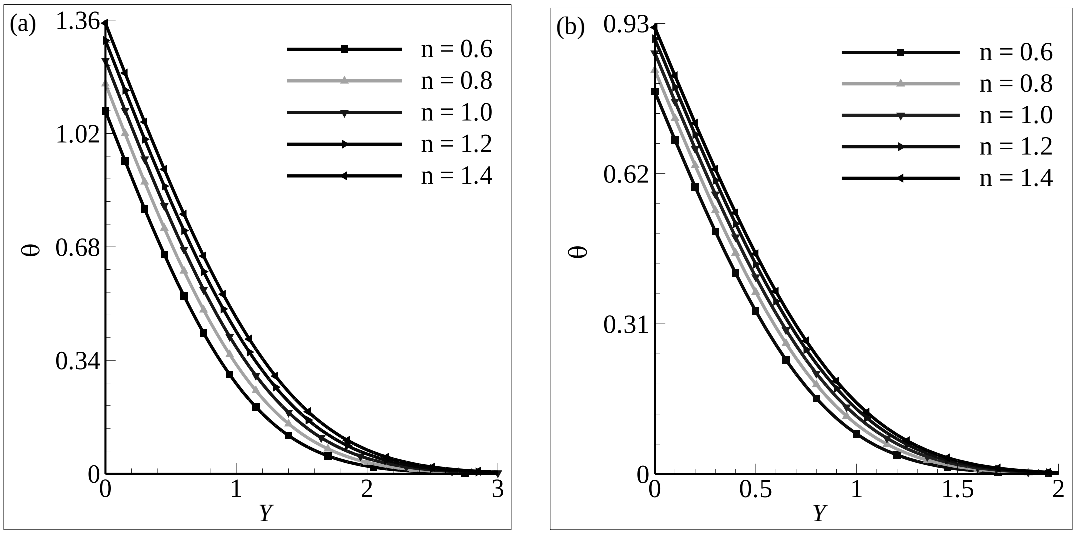

Fig. 5 Temperature distribution for different n at X = 2 for (a) Pr = 10 and (b) Pr = 50.

The effects of the non-Newtonian power-law index n (=0.6, 0.8, 1.0, 1.2, 1.4) on the variation of the wall shear stress $C_f[Gr/4X]^{1/5}$ are shown in Fig. 6a for Pr =10 and in Fig. 6b for Pr =50. The results from these figures clearly show that at the leading edge of non-Newtonian fluids, whose effects start from for

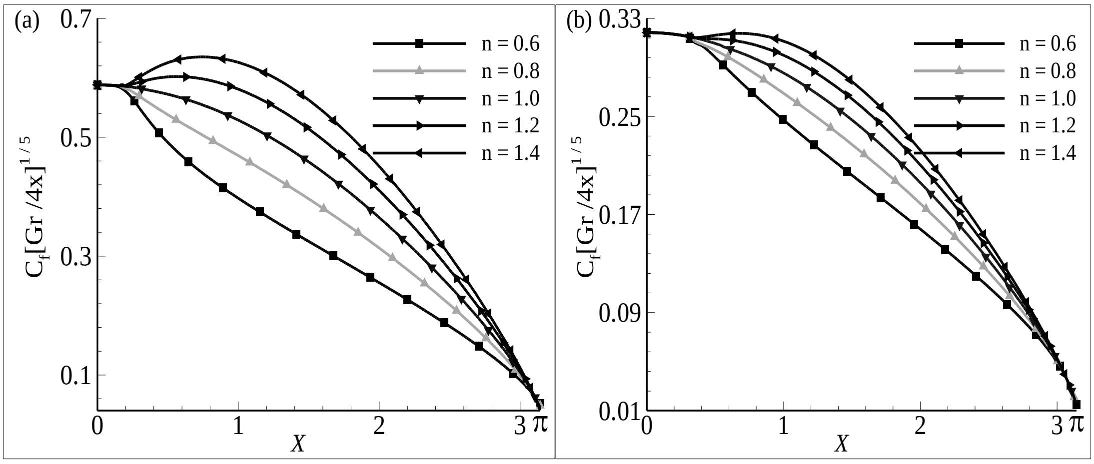

Fig. 6 Wall shear stress for different values of n: (a) Pr =10, (b) Pr = 50.

$X > 0.2$ Pr = 10 and $X > 0.3$ for Pr = 50, the wall shear stress decreases for the shear-thinning fluids (n=0.6 and 0.8) and increases for the shear-thickening fluids (n = 1.2 and 1.4). At the down stream region, there is a similarity solution at $X = 3$ and at $X = \pi$; the boundary-layer of shear-thinning fluids is greater than that of shear-thickening fluids. As expected, the boundary-layer is thinner for larger Prandtl number. Figs. 7(a) and 7(b) represent the local-rate of heat transfer in terms of the local Nusselt number $Nu(Gr/4X)^{-1/5}$ for Pr = 10 and Pr = 50, respectively. The local Nusselt number increases for $n < 1$ and decreases for $n > 1$ at the leading edge of non-Newtonian fluids, whose effects start from

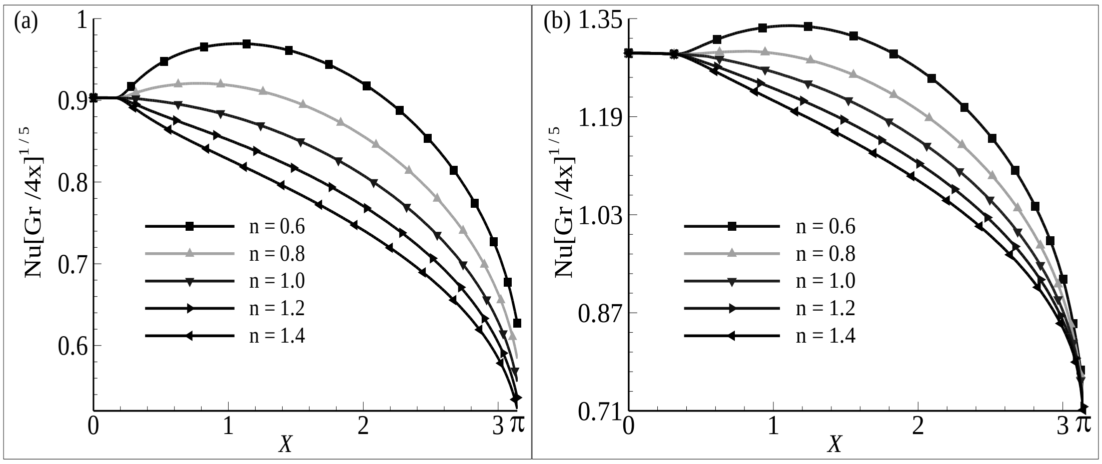

Fig. 7 Local Nusselt number for different values of n: (a) Pr =10, (b) Pr = 50.

$X > 0.18$ for Pr = 10 and $X > 0.32$ for Pr = 50. At the down stream region, heat transfer is similar at $X = \pi$ only for Pr = 50.

4. Conclusions

The proposed modified power-law correlation agrees well with the actual measurements for non-Newtonian fluids; consequently, it is a physically realistic model. The problem associated with the non-removal singularity introduced by the traditional power-law correlations do not exists for the modified power-law correlation proposed in this paper. Therefore, the proposed modified power-law correlations can be used to investigate other heat transfer problems for shear-thinning or shear-thickening non-Newtonian fluids in boundary-layers. The fundamental mechanism is that the effect of non-Newtonian fluids eventually becomes dominant when shear rate increases within the threshold shear limits. We may summarize our obtained results as follows:

- The boundary-layer thickness of the shear rates for Pr = 50 is less than half of the boundary-layer for Pr = 10. All the figures for Pr = 10 are comparatively smooth at *Y* axis than for Pr = 50. At *X* = 3 for Pr = 50, both the fluids (shear-thinning and shear-thickening) are same after the leading edge to down stream regions, but for Pr = 10 at *X* = 3 the fluids are showing different up to down stream regions.
- At the downstream region of the boundary layer, the variation of the temperature inside the boundary-layer is smaller for the case of shear-thinning fluids than that of the shear-thickening non-Newtonian fluids for all Prandtl numbers considered here.

- The boundary-layer thickness decreases more at the downstream region for the shear-thinning fluids than that for the shear-thickening fluids. It is revealed that the boundary-layer thickness for Pr = 50 is almost half of the boundary-layer for Pr = 10.
- It is observed that the local Nusselt number increases when $n < 1$ and decreases when $n > 1$ at the leading edge of non-Newtonian fluids for both Prandtl number considered here.

1. **Nomenclature**

C_f	Local skin-friction
C	Constant
D	Non-dimensional viscosity of the fluid
a	Radius of the circular cylinder
g	Acceleration due to gravity
n	Non-Newtonian power-law index
k	Thermal conductivity of the fluid
C_p	Specific heat at constant pressure
Gr	Grashof number
2. K	Dimensional constant
3. Nu	Local Nusselt number
Pr	Prandtl number
T	Dimensional temperature of the fluid
T_w	Surface temperature of the cylinder
T_∞	Ambient temperature
$\bar{u}, \bar{v}$	Velocity components along the $\bar{x}, \bar{y}$ axes, respectively

4. $\bar{x}$, Cartesian coordinate measured along the surface of the cylinder and normal to it respectively

5. U, V Dimensionless fluid velocities in the *X, Y* directions, respectively

6. X Axial direction along the circular cylinder

7. Y Pseudo-similarity variable

Greek symbols

α Thermal diffusivity

β Thermal expansion coefficient

ρ Fluid density

θ Dimensionless temperature of the fluid

μ Dynamic viscosity

ν (μ / ρ) kinematic viscosity

ν_1 Reference viscosity of the fluid

γ Shear rate

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